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A New Measure of Detail for Triangulated Meshes

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***Abstract*—As the complexity of 3D models used in computer graphics applications grows, there arises a need to visualize the overall distribution of detail on them. Detail is a function of the amount of information present on a surface. In this paper, we present a method to quantify detail using a combination of local measures of curvature and density. We show that detail can be used for applications like ordering for mesh decimation, visualizing abnormalities in a mesh and so on.**

***Index Terms*—Triangular, Meshes, Detail, Density, Curvature, Measure, Visualization, Mesh Deformation, Mesh Decimation**

I. Introduction

As the complexity of 3D models used in computer graphics applications grows, there arises a need to visualize the overall distribution of detail on them. A measure of detail helps us to separate the high frequency information present in a polygon meshes from the large scale structure of the mesh or the low frequency variations. In our paper, we present a method to quantify detail using a combination of local measures of curvature and density.

Detail for a polygon mesh refers to the amount and variation of surface structures. To quantify the variation, measures of curvature exist. Curvature is usually thought of in local terms, i.e., most discrete curvature measures are indicative of variation in a small region around a vertex. To include scale into such a measure so as to have a quantifiable measure of detail, it is necessary to bring in the local density as well. Thus, a measure of detail comes about by combining these two measures in a suitable manner.

After having identified high and low detail regions of a mesh, we can operate selectively on these regions to modify, enhance or remove them. Some direct applications of the above can be ordering of polygons for decimation, creation of mesh aware displacement maps and so on. Such a model can also be used for visualizing the detail in realtime which could be of great use for artists wishing to control the amount of detail present on different regions of a mesh.

II. Related Work

Existing methods for quantifying mesh detail are scarce. However, there have been many papers addressing the issue of local curvature estimation for polygonal meshes utilizing either Gaussian or mean curvature measures.

An early attempt at curvature estimation for discrete surfaces was carried out by Hamann [1] in 1993. A simple scheme for approximating the principal curvatures at each vertex of a triangular mesh is developed using quadratic polynomials. This is based on the fact that a surface can locally be represented as a bivariate function.

A particularly interesting method utilizing differential geometry is presented in the paper by Gelas et al. [2] Here, discrete curvature operators are defined using a combination of two basic operators as expressed by Meyer et al. [3]. These two operators are applied only in what they call the 'region of influence', which is the 1-ring neighborhood of a vertex, making it similar to our approach.

Rusinkiewicz [4] generalizes the discrete curvature measure using a finite-differences approach to better estimate curvatures on irregular triangle meshes. This method is inspired by the simple algorithm to estimate the normal at vertex by taking the weighted average of the normals of faces touching it.

III. Detail Measure

Detail is a function of the amount of information present on a surface. High detail regions have high mesh density as well as high curvature. Regions with high density and low curvature are flat regions which are unnecessarily densely tessellated. Regions with low density and high curvature are large and possibly sharp regions, which may not be represented well.

While local curvature measures can be used to convey a major portion of the mesh detail, they are incomplete in representing it. A detail measure should incorporate a combination of different measures in order to accurately portray the detail on a mesh. In our paper, this is done using a combination of local measures of: a) Density b) Curvature.

A. Density

For density of a mesh, a simple and intuitive measure is the inverse of the average edge length coming out from each vertex. We calculate a density as:

$$\text{Density}(V) = \frac{\text{numNbrs}(V)}{\left(\sum_{i=1}^{\text{numNbrs}(V)} \text{edgeLength}(V, V_i)\right)} \quad (1)$$

Here $\text{edgeLength}(V, V_i)$ is the Euclidean distance between two vertices. V is the vertex for which the density is being calculated and V_i is a first level neighbor of V. $\text{numNbrs}(X)$ is the number of neighbors of the vertex X. In accordance with this measure, when the scale of the model increases, the density decreases because of increase in edge length. Thus,

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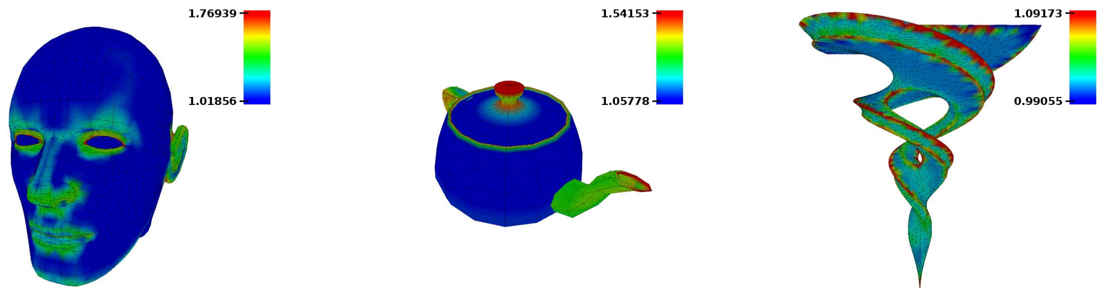


(a) (b) (c)

Fig. 1. Results of visualizing our detail measure on some models; Red areas are regions with high detail while blue regions are those with relatively low detail.

the density is scale dependent. This was chosen because, in an absolute world scale, a model with small edges is considered to have more density (or more triangles per unit area), than a model with larger edge lengths.

It is also possible to make the density measure scale invariant by normalizing the above density using the average edge length of the entire mesh. To do this we can divide the local average edge length by the global average edge length before taking its reciprocal to calculate the density.

B. Curvature

Many definitions exist for curvature. We require one that is fast and numerically bound. We work with the angles between normals in our method. Thus, for every vertex, we find the *cosine difference* between each of its neighboring points normals. This is done as

$$\text{cdiff}(\vec{x}, \vec{y}) = \frac{1}{2}\left[-1 \times \left(\frac{\vec{x}\cdot\vec{y}}{|\vec{x}||\vec{y}|}\right) + 1\right] \quad (2)$$

The curvature for a vertex V with normal $N(X)$ is

$$\text{Curvature}(V) = \sum_{i=1}^{\text{numNbrs}(V)} \frac{\text{cdiff}(N(V), N(V_i))}{\text{numNbrs}(V)} \quad (3)$$

Notice that the cdiff (cosine difference) of any two vectors will always be between zero and one. When the vectors have the same direction, its value is 0 and when they point in opposite directions, it is 1.

C. Detail

Density and Curvature need to be combined to get an overall detail measure. Since curvature is bound between 0 and 1 and density can be any positive number, using addition or multiplication does not produce the best results.

We chose exponent operator since one of our measures was already bound in its range. Thus, using curvature as an exponent to density gave us a desirable combination of the two. We define $\text{Detail}(V) = \text{Density}(V)^{\text{Curvature}(V)}$.

Density and curvature are not completely independent. As the density increases, the curvature will decrease since curvature for a vertex is calculated using its 1-ring neighborhood. Extremely dense regions will have points with normals pointing in the same direction if they are fit to a limiting surface. Thus, as density tends to ∞, curvature will tend towards 0.

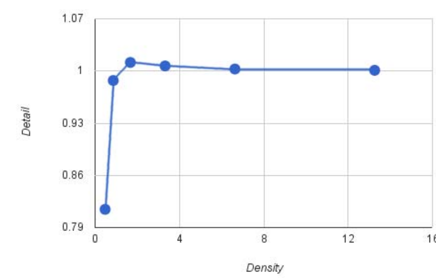


Fig. 2. Variation in Detail as Density increases upon an icosphere of radius 2. We see that the detail initially rises, reaches a peak and then tends to 1 as the density $\rightarrow \infty$, thereby causing curvature $\rightarrow 0$.

We rewrite the equations in terms of the angle θ_i, between the normals $N(V)$ and $N(V_i)$. We can also assume that in the limiting case, every neighboring edge E_i becomes a chord subtending angle θ_i at the center of a circle with radius R_i. The curvature becomes $\sum_{i=1}^{N} \frac{\frac{1}{2}[-1 \times \cos(\theta_i) + 1]}{N}$ and density becomes $\text{Density}(V) = \frac{N}{\left(\sum_{i=1}^{N} 2R_i \sin(\theta_i/2)\right)}$.

When θ_i tends to 0 for all i, we can make the simplifying assumption that every $\theta_i = \theta$ and every $R_i = R$ (all the neighbors and the vertex V are placed on the surface of a sphere with finite radius R such that each edge subtends an angle of θ at the center).

Then the limit of detail

$$\lim_{(\theta\rightarrow 0)} \frac{1}{2R\sin(\theta/2)}^{\frac{1}{2}[-1\times\cos(\theta)+1]} = 1.$$

We see that, as density tends to ∞ and curvature approaches 0, detail tends to 1. We study the variation of density using an icosphere of radius 2 at different subdivision and hence density levels. Figure 2 shows that the detail values corroborate the above results. In Figure 3, we see how, upon changing the scale of an icosphere mesh, the detail varies. We see that as the mesh increases in size, the detail decreases.

IV. Results and Applications

Figure 1 shows some of the visualizations of this measure of detail on various meshes. As we can see in Figure 1(a), most of the mesh has uniform curvature yet density is maximum

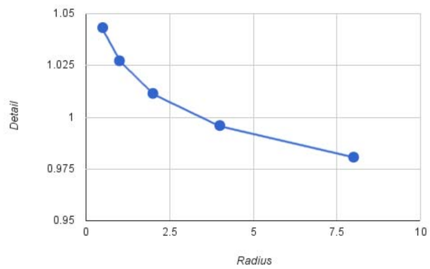
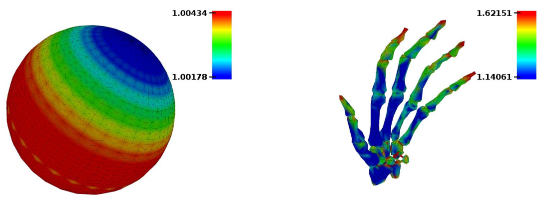


Fig. 3. Variation in Detail as the radius of an icosphere is increased, thereby changing the density but not the curvature.

around the eyelids and this is visualized as red regions. Similarly in Figure 1(c) the mesh is mostly uniform in density yet the curvature changes sharply at the edges and thus those are represented with a high detail value as opposed to the smoother cyan regions. Figures 1(b) and 4 show some more results.



(a) (b)

Fig. 4. More results; Red areas are regions with high detail while blue regions are those with relatively low detail.

Having built up this measure of detail, we now have a way of visualizing detail and having a measure of detail for relative comparison within a polygon mesh. There can be various applications. Some of the possible applications are described below.

A. Mesh Decimation

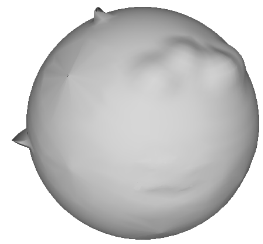
Mesh decimation can be achieved through many existing algorithms such as edge collapse [5], triangle collapse [6]. Current methods to decide which vertices, edges or triangles to remove are based on error measures such as quadric error measures [7] to ensure that the mesh retains most of its shape. Such error measures are usually quite expensive to compute.

An alternate ordering could be found using this measure of detail to sort the vertices/edges/polygons as we typically want high detail regions to be decimated to lower detail regions. We can contain the removal of the detailed regions with a threshold so as to not modify the shape significantly.

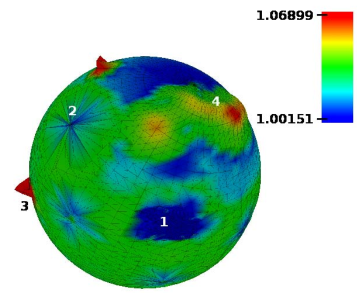
Another goal of decimation of a mesh, when used to create a low resolution mesh along with a displacement map, is to transfer detail from the mesh to the displacement map. For such an application too, this measure can help choose which polygons to decimate so as to have maximum information shifted to the displacement map.

B. Identification of Abnormalities in Mesh Structures

Uniform mesh structures the easiest to work with and provide numerical stability for a multitude of operations such as differential operations or parameterizations. When modelers create polygon mesh models, they are not always able to create uniform or even piece-wise uniform meshes due to the nature of the modeling wherein they emulate the overall shape of the target object without too much consideration for the underlying mesh topology. This is particularly true for organic modeling as opposed to mechanical modeling.



(a)

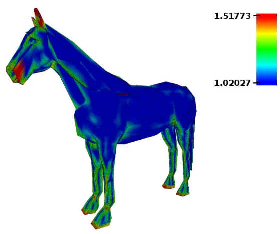


(b)

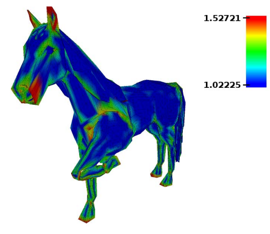
Fig. 5. The first figure shows a model which has abnormalities which are not visible under regular lighting. Upon visualizing the detail, these abnormalities become obvious as shown in the second.

An abnormality in a mesh would usually have at least one of the following properties; A region of triangles significantly different in size from the surrounding triangles; A region which does not ”flow” well with the surroundings. Such a region typically would entail protrusions and depressions. It can be characterized by high curvature values.

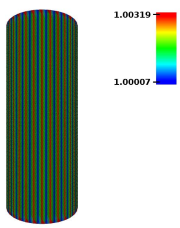
Our measure of detail will capture both these anomalies in many meshes. The first category will refer to outliers in terms of density and thus have either a much larger or lower density value compared to its surroundings. The second category refers to regions having a high curvature value. Thus these regions will be easy to identify visually. In Figure 5, we see a model under regular lighting and see that it appears to be more or less uniform. Upon visualizing the detail in the model, we notice many abnormalities as shown in Figure 5(b). Region 1 appears blue since the density is very high, which in turn causes the curvature to become very small for a smooth curve. Region 2 shows highly skewed triangles which appear as a visual discontinuity. Region 3 shows a protrusion; an abrupt increase in curvature while the density is not changed much



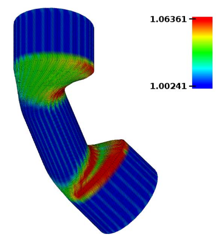
(a) Original Horse Mesh



(b) Deformed Horse Mesh



(c) Original Cylinder Mesh



(d) Deformed Cylinder Mesh

Fig. 6. Visualizing detail on deformed models. Figures 6(b) and 6(d) show the detail measure after deformation has been applied on Figures 6(a) and 6(c) respectively.

and hence appears as red. Region 4 shows bumps in the mesh which show up as red regions as the curvature increases.

This approach can be further extended to incorporate a supervised learning algorithm to identify statistical abnormalities. For example, a support vector machine can be trained on a data set and be used to recognize these abnormalities.

C. Visualization of Magnitude of Mesh Deformation

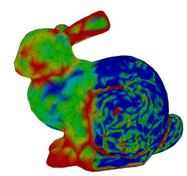
Upon deformation, the affected area of a polygon mesh undergoes twisting and stretching. These manifest as a change in the detail of that region. Thus, we can use our measure to visualize the extent of the effect of the deformation. As seen in Figure 6, upon deformation the detail changes noticeably. In Figure 6(b), the effects of deforming the leg of original mesh in Figure 6(a) can be seen mainly in the front left shoulder region, as well as the elbow. This occurs because the change in the shape of the leg by rigging causes stretching and twisting in these regions, which manifests as a change in detail. In Figures 6(c) and 6(d), the deformation of a relatively high polygon cylinder by twisting and squeezing is visualized. The regions where the cylinder begins to increase in radius show the most deformation, because the middle section of the cylinder is affected more by the twisting than the rest of the cylinder.

D. Detail aware texture mapping

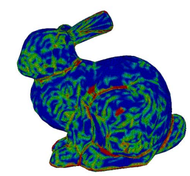
The knowledge of the distribution of detail on a surface can be used by an artist to create texture maps appropriate to the local detail level. That is, low detail regions can be given higher resolution textures to make up for the low information content of these regions and vice versa. This can be done manually, or it can be automated. UV coordinates can be generated automatically, by following the simple rule that the texture resolution is inversely related to the detail. These UV coordinates can then be used for normal texture maps, bump maps, displacement maps and so on.

E. Uniformization of mesh detail

For any given mesh, there will be variations in detail, unless the mesh is regular (for example a regular polyhedron). Uniformity of a mesh is a desirable quality for many purposes, both algorithmic and artistic. Therefore, once the detail information for a mesh is known, selective subdivision can be performed on regions with relatively lower detail, in proportion to the amount of detail, such that the mesh gains uniformity of structure. Once again there exists an inverse relationship between the level of subdivision and the detail. An example can be seen in Figure 7. As we see in Figure 7(a), the left half of the mesh is of high detail compared to the right half. Upon subdivision of the left side, in accordance with the high detail, the mesh becomes uniform and hence the detail is evenly distributed as shown in Figure 7(b).



(a) Non Uniform Detail



(b) Mesh after uniformization

Fig. 7. Detail Uniformization

V. Conclusion

We have defined a scale dependent detail as a function of density and curvature. This detail measure helps identify high and low frequency regions on the surface of a mesh. We also show how detail can be used for a variety of applications such as detail aware texture maps, visualizing abnormalities in a mesh, ordering for mesh decimation.

References

[1] B. Hamann, *Curvature approximation for triangulated surfaces*. London, UK: Springer-Verlag, 1993, pp. 139–153.

[2] G. A., G. A., and M. S., "Surface mesh discrete curvature estimators," in *The Insight Journal*, 2008.

[3] M. Meyer, M. Desbrun, P. Schröder, and A. H. Barr, "Discrete differential-geometry operators for triangulated 2-manifolds," in *Visualization and Mathematics III*, H.-C. Hege and K. Polthier, Eds. Heidelberg: Springer-Verlag, 2003, pp. 35–57.

[4] S. Rusinkiewicz, "Estimating curvatures and their derivatives on triangle meshes," in *Proceedings of the 3D Data Processing, Visualization, and Transmission, 2nd International Symposium*, ser. 3DPVT '04, 2004, pp. 486–493.

[5] L. Kobbelt, S. Campagna, and H. peter Seidel, "A general framework for mesh decimation," in *in Proceedings of Graphics Interface*, 1998, pp. 43–50.

[6] P. Zhigeng, S. Jiaoying, and Z. Kun, "A new mesh simplification algorithm based on triangle collapses," *J. Comput. Sci. Technol.*, vol. 16, pp. 57–63, January 2001.

[7] M. Garland and P. S. Heckbert, "Surface simplification using quadric error metrics," in *Proceedings of the 24th annual conference on Computer graphics and interactive techniques*, ser. SIGGRAPH '97, 1997, pp. 209–216.